butes to quadratic or linear behavior depends on the magnitude of σ_i/H, where σ_i is a measure of internal strain. If H is sufficiently large, this quantity will be small for all σ_i and quadratic behavior observed, while if H is sufficiently small, many local strain regions will contribute to linear behavior and the linear term will dominate.

(ii) Similar variation of a and b with increased internal strain, as observed by Parfenov and Voroshilov, is expected since local strain regions contributing to both linear and quadratic behavior would increase.

(iii) Parfenov and Voroshilov also observed that a is proportional to M_s under temperature variation in nickel. From this calculation, a is proportional to $B/\mu M_s$ as shown by Eq. (13), and since, in nickel, B/μ is proportional to M_s^2,[15] this behavior is expected.

VII. CONCLUSION

The primary conclusion is to suggest that the a/H term in the expression for the approach to saturation has been overemphasized. Its origin is in the residual internal strain of magnetic material and it has validity only over a limited region of the H axis. Secondary results are the model and calculation which determine the magnetic behavior of porous magnetic material subject to hydrostatic pressure. It is worth mentioning that this technique suggests a method for controlled investigation of the effects of internal strain on material properties.

ACKNOWLEDGMENT

The author wishes to thank Professor G. E. Duvall for his helpful discussions during the course of this work.

APPENDIX

The following calculation will show that the subsequent functional dependence of M/M_s on P/H, after the initial quadratic behavior, is linear with a slope given by Eq. (13). The magnetic equilibrium relation, Eq. (9), can be written

$$\frac{r^3}{a^3}=\frac{3}{4}\,\frac{\sin 2(\psi+\theta)}{\sin\Psi}\,\frac{BP}{\mu M_s H}\ . \tag{14}$$

This, in principle, can be solved for $\cos\psi$, giving

$$\cos\psi = g\left(\frac{u^3}{P/h}\,,\,\theta\right),$$

where g is an unknown function, $u=r/a$, and $h=\mu M_s H/B$ is the reduced field. Averaging $\cos\psi$ over a spherical surface gives

$$(\cos\psi)_{\rm av}=m(u)=k\left(\frac{u^3}{P/h}\right),$$

where k is another unknown function and $m(u)$ is the average normalized magnetization in the direction of the applied field in a spherical shell at a radius r. This can be inverted to obtain

$$u^3=(P/h)f(m)\ . \tag{15}$$

Again f is unknown. Equation (15) will be used in the following. First an expression for the macroscopic magnetization in the porous material is required. In terms of the proposed model in Sec. II this is

$$\frac{M}{M_s}=\frac{4\pi}{\frac{4}{3}\pi r_0^3}\int_a^{r_0} m r^2\,dr$$

or

$$\frac{M}{M_s}=3p\int_1^{r_0/a} m u^2\,du\ ,$$

where $p=a^3/r_0^3$ is porosity.

In anticipation of linear behavior consider

$$\frac{dM/M_s}{dP/h}=3p\int_1^{r_0/a}\left.\frac{\partial m}{\partial P/h}\right|_u u^2\,du\ .$$

The mathematical identity

$$\left.\frac{\partial m}{\partial P/h}\right|_u=-\left.\frac{\partial u}{\partial P/h}\right|_m\left.\frac{\partial m}{\partial u}\right|_{P/h}$$

with Eq. (15) gives

$$\left.\frac{\partial m}{\partial P/h}\right|_u=-\frac{f(m)}{3u^2}\left.\frac{\partial m}{\partial u}\right|_{P/h}\ ,$$

and therefore

$$\frac{dM/M_s}{dP/h}=-p\int_1^{r_0/a} f(m)\left.\frac{\partial m}{\partial u}\right|_{P/h} du\ .$$

In a region where the magneto-elastic energy dominates at the lower integration limit while the magnetic energy dominates at the upper limit, the integral transforms to

$$\frac{dM/M_s}{dP/h}=-p\int_{\pi/4}^{1} f(m)\,dm\ . \tag{16}$$

This shows the anticipated linear behavior which is expected to occur in some region of the P/h axis. Equation (16) is Eq. (13) with γ given by the integral expression.

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*Research sponsored by the U. S. Air Force Office of Scientific Research under Contract No. AFOSR-69-1758.

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